The Rating Game

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Abstract

Ever wonder how squash or tennis rating systems work? Are you interested in designing one? If so, this is the problem for you. We will give you the algorithm for determining the rating changes of two players after they have played a match: you have to design the system to keep the ratings up to date.

Sounds easy? It is, if match results are input in chronological order. However, the real world does not work that way! Players enter results out of order and make mistakes. The system has to maintain the ratings as if the corrected match results were input in the correct order. Not so easy.

Players also want to track the effects of each match on their rating, find challenging opponents (particularly if they are improving quickly) and see their historical performance against a particular opponent. Club pros also get into the game (pun intended) to set initial ratings, to adjust ratings, and to enter or fix match results for members of their clubs.

Domain Description

Squash is a racket sport played in an indoor court by two players. Play alternates by each player trying to hit the ball according to prescribed rules so that the other player cannot make a legal return shot. The last person to make a legal return shot in the rally starts the next rally and is awarded a point if he started the rally.

Definitions:

> **Game**: A game ends when one player wins 9 points.

> **Match**: A match consists of up to five games but ends as soon as one of the players wins 3 games. Thus a match is "best of five".

A match result is the record of the number of games won by each player; that is one of 3-2, 3-1, 3-0, 0-3, 1-3, or 2-3.

The Required Program

The local squash club wants a web based application for keeping its player's ratings up to date. A player's rating after a match, which depends on his own and his opponent's current rating and on the match result, should be recalculated after each match played.

If match results are entered in chronological order, then this simply means calculating the new player ratings as each match is entered. If match results are entered out of order, or if an existing match result is corrected, then the system has to maintain the ratings as if the correct match results were input in the correct order.

Assigned Ratings

All players that exist in the system have an initial **assigned rating** with a date/time stamp of when the player joined the club. If a player's skill level becomes inconsistent with the rating in the system (e.g. due to great improvements or after returning from an injury), then the club pro is allowed to enter a new assigned rating with a new date/time stamp. The system is not allowed to change assigned ratings.

Player rating at time T

A player's rating at time T is the more recent of:

1. the player's most recent assigned rating before time T; or
2. the player's rating resulting from the most recent match played prior to time T.

Note that a player cannot play two matches at the same time. Also, to avoid ambiguity, an assigned rating for a player may not be at the same time as any of that player's matches.

Rating Relationships

The rating of each player resulting from a match is dependent on the rating of both players at the time of the match. This usually means that it is dependent on the results of the previous matches of both players.

If you follow these dependencies backwards in time it means that the ratings resulting from a match depend on the ratings of opponents that these two players have played in the past, and the ratings of their opponents, and their opponents' opponents, and so on.

From the forward-looking perspective, any matches played in the future by these two players, or their future opponents, or their opponents' future opponents, and so on, will depend on the ratings resulting from this match.

A useful definition at this point is:

> A **dependent match** is a match played after the assigned rating or subject match where the dependent match involves the player of the assigned rating or either player of the subject match. This is a transitive relationship.

As an example, suppose Ralph and Gail join the club and play their first match on Sunday, then Gail plays Henry on Tuesday, and Henry plays George on Wednesday. We have the following dependencies:

> Ralph v. Gail is not dependent on other matches
>
> - although it is dependent on their initial assigned ratings
>
> Gail v. Henry is dependent on Ralph v. Gail; and
>
> George v. Henry is dependent on Gail v. Henry

and through transitivity:

> George v. Henry is dependent on Ralph v. Gail

even though neither George nor Henry played in the Ralph v. Gail match. There may be other dependencies not mentioned, such as Gail v. Henry being dependent on the last match that Henry played.

The consequence of this is that if the match result of Ralph v. Gail is edited, then the ratings of all players in all dependent matches need to be recalculated.

This leads to another useful definition:

A **dependent rating:**

1. of an **assigned rating** is the rating for any player that must be recalculated because the player was the subject of the assigned rating or was a player in a dependent match;

2. of a **match** is the rating for any player that must be recalculated because the player played in the match or was a player in a dependent match.

The Rating Formula

The new rating for Player A resulting from a match against Player B is calculated according the following formula:

$$R'_A = R_A + K(S_A - E_A) + w_A - l_A$$

Where:

R_A is the rating of Player A before the match

w_A is the number of games won by Player A in the match

l_A is the number of games lost by Player A in the match

K is 32 if $R_A < 1500$,

16 if $R_A > 1900$, and

24 otherwise.

S_A is 1 if player A won 3 games in the match, and 0 otherwise

and the expected score of player A is:

$$E_A = \frac{1}{1 + 10^{(R_B - R_A)/400}}$$

Similarly, Player B's resulting rating is calculated using this formula with "A" and "B" interchanged.

Note that $E_A + E_B = 1.0$.

Actors

The Rating Game application has two primary actors:

1. squash player; and
2. club pro.

Squash players will use the system to enter, edit and delete matches, to view match results, and to find other similarly ranked players. Club pros will use the system to add players, assign ratings, edit and delete ratings (and do anything a player can do on behalf of the player). Below are the specific user stories for each primary actor.

User Stories

This section describes the main success scenario of the user stories for each actor (following the guidelines in [3]). The scope for all the user stories in this document are the Rating Game web site application, and are written at the "user goal" level. Each story also has as a precondition that the user is logged in and has the privileges needed to perform the story. There are implicit extensions to each story for handling validation errors by displaying an appropriate error message and allowing the user to change or abandon the input. Similarly, most stories should display a message indicating that the requested action completed successfully.

User Stories for a squash player:

Enter a match result

1. player chooses an opponent,
2. player enters the match result and the date/time
3. system validates the input including, but not limited to:
   (a) the date/time is not to be before the initial assigned rating of either player
   (b) the date/time is not to be the same as another match or assigned rating for either player
4. player saves the match and the system updates all dependent ratings

Edit a match result

1. player selects a match to edit
2. player changes the match result or date/time or both
3. system validates the input including:
    a) using the same rules as for entering a match
4. player saves the match and the system updates all the formerly and newly dependent ratings

   Hint: first consider what happens if the match changed to an earlier time, then consider if it changed to a later time.

Delete a match

1. player finds a match
2. player deletes the match and the system updates formerly dependent ratings.

View similarly ranked players

1. View a table containing the following columns for each opponent:
   a) rating
   b) name
   c) contact information (out of scope for DesignFest)

View recent match results

To track the effects of match results on their ratings:

1. View a table containing the following columns for each match recently played:
   a) rating of player at time of the match

b) games won by the player

c) games won by the opponent

d) rating of opponent at time of the match

e) name of the opponent

f) the date and time of the match

For a club pro:

A club pro can enter, edit and delete matches on behalf of any player in the system. In addition, the club pro adds players, updates players' names, assigns ratings, updates ratings and deletes ratings as described in the following user stories.

Add a player

1. club pro enters player name and initial rating
2. system validates input including, but not limited to:
    a) name is not to duplicate another player's name in the system
    b) rating is not to be outside the acceptable range
3. club pro saves the new player

Assign a new rating

1. club pro selects a player
2. club pro inputs the rating and the date/time
3. system validates input including, but not limited to:
    a) rating is not to be outside the acceptable range
    b) time/date is not be the the same as a match or another assigned rating for the player
4. club pro saves the assigned rating and the system updates the dependent ratings

Edit an assigned rating

1. club pro finds an assigned rating to edit
2. club pro changes the rating or the date/time or both
3. system validates input
    a) using the same rules as for assigning a new rating
4. club pro saves the changes and the system updates the formerly and newly dependent ratings

Delete an assigned rating

1. club pro finds the assigned rating to delete
2. club pro deletes the rating and the system updates the formerly dependent ratings

View all players

1. club pro selects view to see all players and their ratings, sorted by rating, with the following columns:
    a) player rating

    b) player name

    c) player contact information (out of scope for DesignFest)

Interfaces

No interfaces to external systems.

What if?

1. Consider what would happen if there was a system failure while recalculating a set of dependent ratings.

2. What would happen if entering or changing a match or assigned rating caused a dependent rating to go outside of the acceptable range for ratings?

3. What would happen if the club pro changed the date of an initial rating to after the date of the player's first game? What should happen?

4. What if matches can now be best of 3 or still best of 5?

5. What if you need to modify the ELO formula?

References

[1]   http://en.wikipedia.org/wiki/ELO_rating_system

[2]   http://www.uschess.org/about/forms/01ratingsystemnew.pdf

[3]   Alistair Cockburn, "Writing Effective Use Cases", Addison-Wesley, 2001